

**Unit 3a: Proportion and Scale**

We just talked about solving proportions in the last unit, so it should be fresh in your mind. Remember that the tried-and-true way of solving a proportion is to

Cross-multiply + divide.

Example:

$$\frac{3}{10} = \frac{x}{670}$$

$$\frac{3 \cdot 670}{10} = \frac{10x}{10}$$

$$x = 201$$

In this unit, we will apply our proportions to real-life situations – mostly geometric figures. This means that we have to care about UNITS!

Let's step back and talk about **ratios and rates**. These are pretty similar, so we will be a bit careless and use them interchangeably in this unit – but usually, ratios will be two things that have the same units. Let's translate the following ratios and rates into math:

Quantity	Written as a ratio	Written as a fraction	Reduced (if necessary)	
3 students out of every 12	3 : 12	3 / 12	1 / 4	Units are students/students, so we can drop them
\$5.00 for 4 cans	\$5.00: 4 cans	\$5.00/4 cans	\$1.25/can	Keep the units since they're different – notice you end up with a unit rate
8 mm every year	8 mm: 1 yr	8 mm/yr	8 mm/yr	
\$0.94 per litre	\$0.94: 1 L	\$0.94/L		
92 out of 600 households	92/600	92/600	0.15	(Round to what seems reasonable)
50% of everyone here	50: 100	50/100	0.50	

We set up a proportion to compare ratios or find a unit rate:

Ex. If 12 students out of 32 in this class have a dog, how many students out of 670 have a dog?

$$\frac{12 \text{ students}}{32 \text{ students}} = \frac{x}{670} \Rightarrow \frac{12 \cdot 670}{32} = \frac{32x}{32}$$

$$x = 251$$

By the way, this also is how you do percentages, whether you know it or not:

$$\frac{17}{26} = \frac{x}{100}$$

$$\frac{17 \cdot 100}{26} = \frac{26x}{26}$$

$$x = 65$$

$\Rightarrow$

$$65\%$$

Ex. It took me 2.4 hours to drive 192 km. What was my average speed?

Unit rate: km/hr

$$\frac{192 \text{ km}}{2.4 \text{ hr}} = \frac{x \text{ km}}{1 \text{ hr}}$$

$$\frac{(192 \text{ km})(1 \text{ hr})}{2.4 \text{ hr}} = x$$

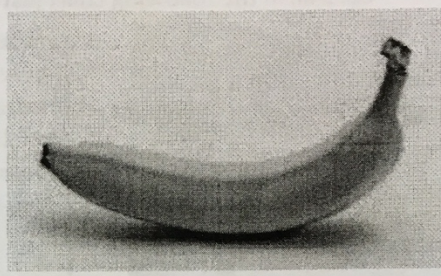
$$x = 80 \text{ km/hr}$$

Name: Key

Guided notes 3a.2  
Math 9 – Wolfe

You can use proportions to reduce or enlarge something as well. Say you want to make a giant statue in homage of your favourite fruit, the banana:

So you measure the length of the banana and it is 21 cm. You want your statue to be 3.0 m long, so you determine the **scale factor**. It's easiest to work with if your units are the same for the actual item and the model, so let's use



3.0 m = 300 cm.

The scale factor is: 300 cm compared to 21 cm

Or  $\frac{300}{21} = \frac{100}{7}$       Or even:  $\sim 14.29\%$   
 $100 : 7$        $14.29 : 1$

Once you know your scale factor, you can calculate all the dimensions of your banana sculpture by measuring the real thing:

	Real measurement	Calculations	Statue
Length	21 cm	$\frac{300 \text{ cm}}{21 \text{ cm}} = 14.29$	300 cm
Circumference at centre	13 cm	$\frac{\text{New}}{\text{Old}} = \frac{x}{13} = \frac{300}{21}$ $\frac{21x}{21} = \frac{300 \cdot 13}{21}$	186 cm
Length of stem	3.8 cm	$3.8 \text{ cm} \times 14.29 =$	54.3 cm

2 ways to do it

And now you're ready to sculpt your monument to the mighty banana!

The scale factor always compares Replica to original, or new to old. If it gets bigger, we call that an Enlargement. If we make a smaller replica, it's a reduction.

Let's fill in the following chart just to practice calculating scale factors:

Original	Replica	Calculations	Scale factor	Reduction or Enlargement
14 inches	7 inches	$\frac{7 \text{ inches}}{14 \text{ inches}} = \frac{7}{14} = 0.5$	0.5	R
0.54 m	4.26 m	$\frac{4.26 \text{ m}}{0.54 \text{ m}} = 7.9$	7.9	E
1 cm	2500 m	$\frac{2500 \text{ m}}{1 \text{ cm}} = \frac{250000 \text{ cm}}{1 \text{ cm}}$	250,000	E
4" x 10"	6" x 15"	$\frac{6}{4} = 1.5$ ✓ both the same $\frac{15}{10} = 1.5$	1.5	E
16 m	48 m	$16 \text{ m} \times 3 = 48 \text{ m}$	3	E
19.2	4.80 cm	$\frac{4.80 \text{ cm}}{0.25} = 19.2 \text{ cm}$	0.25	R

↑  
If you can't remember whether to  $\times$  or  $\div$ , set up a proportion:

$$\frac{\text{New}}{\text{old}} = \frac{4.80 \text{ cm}}{x} = \frac{0.25}{1}$$

$$\frac{(4.80)(1)}{(0.25)} = x$$